NATURAL FREQUENCIES OF HULL VIBRATION

U.S. Experimental Model Basin
Navy Yard, Washington, D. C.

July 1930. Report No. 263.

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I. SUMMARY.

1. The best way of eliminating troublesome vibrations is to remove their source. However, this cannot always be done and accurate estimates of natural hull frequencies would be useful in design as engine frequencies might then be kept clear. A more important reason for improving our information about natural hull frequencies arises from the data thus obtainable on actual elastic characteristics. In this paper results of tests made on a number of Naval vessels are reported, and methods of calculation discussed which are designed to permit prediction of natural frequency. In all the cases included, vibration is in the vertical plane in the fundamental mode with two nodes only.

2. Static tests on U.S.S. Cuyama, a fleet oiler, show that deflections are about 4/3 times values calculated by use of the sectional moment of inertia experimentally determined. This discrepancy is due to local stress concentrations, slip in rivetted joints, buckled plating, and similar causes. Although not strictly justifiable in general, for purposes of dealing with hull vibration it is convenient to associate with the value of E, the modulus of elasticity, an empirical coefficient which in this case would be 75%. In

round numbers the effective modulus of elasticity would then be 10,000 $\frac{\text{tons}}{\text{in.}^2}$ and this is the value used throughout in what follows.

3. The simple formula for the frequency of a bar of uniform section is

$$n = \frac{4.730^2}{2\pi}\sqrt{\frac{gEI}{DL^3}}$$

g is the acceleration of gravity in ft./sec.^{2}, E is Young's Modulus in tons per square inch, I is sectional moment of inertia in inch^2 foot^2, D is displacement (total weight) in tons, L is length in feet, n is frequency per second. The radical being of the same dimensions as n, the constant is dimensionless and equal to 3.56.* The quantity under the radical is in units equivalent to absolute, since the inches and the weight units cancel out.

*Note.- It is customary in England to combine gE with the dimensionless constant and express frequency in cycles per minute. The conversion factor is

$$60\ \frac{\text{sec}}{\text{min}}\sqrt{32.15\ \frac{\text{ft}}{\text{sec}^2} \times 10{,}000\ \frac{\text{tons}}{\text{sq.in.}}} = 34{,}000$$

and the formula becomes

$$N = 121{,}000\sqrt{\frac{I}{DL^3}}$$

I being in inch^2 foot^2, D in tons, L in feet and N in cycles per minute.

For discussion of Schlick's formula and constant see Section 8 below.

II. OBSERVATIONS ON FOUR OILERS.

4. Natural frequency was determined experimentally by means of a Sperry Pallograph, the ship being set into vibration by dropping an anchor through a scope of four or five links, bringing it up sharply on a chain stopper. Results obtained were as follows:

TABLE I.

Ship	Frequency per min.	Displacement tons	EI #ft.^{2} x 10^{-12}	Schlick's constant absolute
Cuyama	60.3	15,430	*9.2	3.29
Brazos	88.5	7,600	10.0	*3.29
Neches	81.1	12,600	14.0	*3.29
Salinas	73.5	10,000	9.1	*3.29

*Value accepted for application of Schlick's formula.

The value of EI was obtained from static tests on the Cuyama and Schlick's constant inferred. On the other vessels this process was reversed; the type of ship being the same, the constant presumably has the same value for all, which permits determining EI by the simple test described.

The natural frequency on each of these vessels differs radically from the engine speed at maximum of hull vibration, indicating that unison between the engines and the hull is not the controlling circumstances in determining this critical speed.

III. OBSERVATIONS ON THE OKLAHOMA.

5. In the case of the OKLAHOMA the amplitude of vibration is much less than on the CUYAMA. It occurs at speeds close to the natural frequency, and it seems perfectly clear that it is due to unison between the unbalanced forces proceeding from the propelling system and the natural frequency of the hull. The two-node character of the vibration is well-marked.

The report of increased difficulties due to vibration about the bridge and on the masts since modernization may be explained as follows: The masts are rather close to the nodal points, at which rotational oscillations occur though vertical motion there is zero. The new tripod masts, being much stiffer than the cage masts, operate to magnify this effect, giving a horizontal vibration in the fore and aft direction, which is actually observed in the fire control top.

At certain times this vibration might interfere somewhat with use of instruments. If so, the best solution, apart from counterbalancing the engines to eliminate the source of trouble, would lie in developing a suspension for the instruments involved which would act, as the cage mast did, to absorb these vibrations. No alterations to the hull could possibly do more than shift the maximum to a slightly different speed.

6. The tests consisted of three parts: First, the ship was operated at a series of speeds ranging by increments of 3 r.p.m. from 115 to 64. This located the critical speed approximately. The region about the critical speed was then explored in speed increments of 1 r.p.m. from 80 to 86 and the maximum located more exactly very nearly at 82. Next, the engine was set to give maximum vibration and held there while the amplitudes were explored along the whole length of the ship.

The results of these tests are shown in Figures 1 and 2, which give the variation of amplitude with speed at a fixed station, in the chart house, and the variation of amplitude along the length of the ship at fixed speed. This locates the nodes approximately.

Finally, the natural frequency of the ship was determined, with engines stopped, vibrations being started by an impulse from an anchor dropped and brought up sharply on a chain stopper. Although the two types of recording instruments used failed to agree as to the frequency of the resulting vibration, the discrepancy is not great enough to cause any doubt that the critical speed at about 82 r.p.m. is due to unison between engines and hull.

IV. DETAILS OF THE OKLAHOMA TESTS.

a. Two Sperry pallographs were used, and a Geiger vibrograph was also available for the frequency determinations. Independent data were taken by a party from the Navy Yard, New York, with the Geiger instrument on length-distribution of amplitude, but these were to be separately reported to the Bureau of Engineering.

During the first stage of the tests one of the pallographs was at a fixed station amidships, during the second it was moved at intervals of 10 frames from bow to stern. The other pallograph was in the chart house, and simultaneous records were made to permit elimination of irregular time fluctuations in amplitude. For the frequency test one pallograph was right forward and one right aft. The Geiger instrument was placed amidships for the first trial, but as amplitude there was found insufficient, it was taken right forward and placed in the same compartment with the pallograph for the second and third frequency trials.

The values for natural frequencies obtained were as in Table II.

The discrepancy between the results obtained from the two types of instrument is too great to be considered accidental. Although the value obtained from the Geiger type lies somewhat nearer the engine speed of maximum hull vibration it is not conclusively shown that this is a more correct value

under the conditions attending the natural frequency test. In both instruments the time control left something to be desired. On the Sperry record tape 5 second intervals from watch were marked by hand. On the Geiger tape a vibrating spring left a record at intervals of about one second, but the record was not continuous. Motion of the vibrating system was much more strongly damped in the Geiger instrument. Free period of Sperry was about 3 seconds, Geiger unknown. Tests of both instruments at known frequencies should be made to obtain definite calibration data.

As matters stand, determination of natural frequency of vibrations following a single exciting impulse at one end of the ship must be considered subject to an error of about 10%; measurement of this important quantity with greater precision must await further refinement of methods.

V. METHODS FOR CALCULATING NATURAL FREQUENCY.

8. A ship differs from a bar suspended at 2 nodes in the following respects:

a. Its weight is not uniformly distributed over its length.

b. Its sectional moment of inertia varies along its length.

c. The structure is loaded with weights which make little or no contribution to the elastic action.

d. The water in which it floats takes part in the motion and so modifies the inertia involved.

In addition the simple formula quoted neglects the effect of

e. Deflection of the bar in shear.

f. Rotational motion, such as occurs especially at the nodes.

About 40 years ago it was proposed by Schlick to lump all these effects into a single factor which could be combined with the absolute constant to form an empirical constant; and for it he quoted from his own experience the following values:

Torpedo Boats	4.21
Liners	4.65
Freighters	3.80

Although this empirical constant should not vary much for vessels of similar type, Schlick's formula has never been extensively applied, in fact the natural frequency of U. S. Naval Vessels is for the most part not ascertained.

A step forward may be taken by correcting Schlick's formula for non-uniformity, leaving the empirical constant to carry only the burden of the other errors mentioned. Several papers on this subject have appeared recently* effort in all being directed toward devising a practical method

*Tobin TINA 1922

Nicholls TINA 1924

Taylor NECI 1927-28

Lewis NAME 1927, 1929.

of applying to ships the calculations originally made by Lord Rayleigh in his Theory of Sound. Results are set forth below in sufficient detail to permit direct application to calculation of natural frequency in course of design; but evaluation of the empirical constant calls for experimental determination of numerous actual frequencies.

9. The underlying thought is that a vibrating ship, when at the phase of extreme departure from its form at rest under no load, may be treated by usual procedure in strength calculations, the load being taken as 100% dynamic. The momentary elastic condition of the ship is identical with that under a static load whose amount is found by multiplying each mass involved by its acceleration. Now of course the accelerations in vibratory motion vary with amplitude of vibration as well as with phase. An arbitrary assumption is made as to amplitude which will later divide out; this is justified for the present by the fact that frequency does not depend on amplitude, so that the final result will not depend on the arbitrary assumption made. This applies primarily to variations of amplitude with time, - as to variation along the length of the ship, that depends on the mode of vibration, and is particularly affected by the weight distributions.

To calculate the dynamic load we must multiply each mass by the product of the assumed amplitude and the acceleration produced by unit deflection from the position of rest. This last factor forms the objective of the whole

procedure, for when we have it, the frequency is equal simply to its square root divided by 2π. To see this, consider the simple harmonic motion as the projection of a uniform circular motion on unit radius with peripheral speed v. The maximum acceleration is v^2 and the number of turns per second is $\frac{v}{2\pi}$. Since frequency is the same all along the ship's length, the acceleration per unit deflection must also be uniform. Introducing this factor, called φ and at first undetermined, we carry through a standard strength calculation which leads to values of deflection, still containing, however, the undetermined factor φ. These must be identical with the assumed amplitudes that formed the starting point, and by division φ is evaluated.

The only questionable point remaining is this: is the distribution of amplitudes originally assumed one that is possible? The answer lies in the values of φ obtained. If they show acceptable uniformity, the assumed distribution of amplitudes sufficiently approximates a possible form.

This curve of distribution of amplitudes is of course identical in form with a curve of deflections under the given load. For the type of vibration now being considered it is chiefly a question of location of the nodes and relative deflection at ends and amidships. For a first approximation the curve for a uniform bar may be taken. Closer approximation could be obtained by adjusting the curve as necessary to obtain uniform values of φ.

The process of passing from a curve of loads to one of deflections involves four integrations leading in turn to curves of shear, bending moment, slope, and deflection. The first may involve readjustments to obtain zero end values. In the third the constant of integration is chosen so as to produce a symmetrical variation of slope from a negative maximum at one end to a positive maximum at the other. The final integration then leads to a symmetrical curve of deflection.

10. In the Transactions of the Society of Naval Architects and Marine Engineers for 1927, Professor Frank M. Lewis gave a detailed procedure for calculating natural frequency which shortens the work considerably in case it is permissible to assume that the curve of amplitudes may be shifted to obtain adequate agreement with the deflection curve by making only those changes which are necessary to close the shear and bending moment curves. Results obtained by applying this method to the OKLAHOMA, which is quite symmetrical, and the CUYAMA, which is rather unsymmetrical, are given below.

Correction for virtual mass due to the inertia of the water has also been studied by Professor Lewis and the results obtained by application of the method described by him in 1929 are also given below.

VI. RESULTS OF FREQUENCY CALCULATIONS.

11. From observations of natural frequency and effective value of Young's Modulus made on the CUYAMA, combined with value of sectional moment of inertia taken from Bureau design data, and approximately confirmed by test, the constant in Schlick's formula is 3.29; this may be regarded as the experimental value. For comparison the theoretical value for a uniform bar, $E = 30 \times 10^6 \frac{\#}{in.^2}$, is 3.56. By the method of calculation exhibited in the appendix, with the non-uniform distribution of weight and section moment of inertia accounted for, a value of 4.47 is found. Application of the shorter method of Lewis (NAME 1927) gives 4.54. The frequency is naturally increased by concentration of weight amidships, and whatever agreement there is between the experimental value of Schlick's constant with that for a uniform bar is due to compensating errors.

In the case of the OKLAHOMA, the value of frequency obtained from the Sperry pallograph, 77.1 per minute, is used, as this is considered comparable with the value found by the same method on the CUYAMA. The moment of inertia having been increased by modernization, the old value of 1,380,000 $in.^2$ $ft.^2$ no longer applies. An approximate new value of 1,818,000 has been obtained by adding 80# of deck armor, blister plating, 3 angles and bilge keel. If a more exact value is necessary, it is preferred that it be obtained in the Bureau.

$E = 10,000 \frac{t}{in.^2}$ and displacement from draughts is 32,600 tons. Schlick's constant, experimental value, is found to be 4.19. The value calculated by the procedure described above is 4.40, and by Lewis' method: 4.44. For easy comparison, these values are exhibited in Table III, together with the results of correction for virtual mass.

12. Of the various sources of error in the calculated results quoted, that due to virtual mass of the water is the greatest. Correcting by the procedure given by Lewis, NAME 1929, gives the values quoted in Table III. In the case of the CUYAMA, fairly close agreement with the experimental value is thus obtained. In the OKLAHOMA it is probable that the effective value of E used, 10,000 t/in.^{2}, is too low, but the discrepancy is too great to be explained in this way alone. Comparison of the calculated values in Table III with the experimental values gives the factor which must be applied to account for effects of inertia of water, deflection in shear, rotational energy, and departure of effective value of E from the assumed value, 10,000 tons/in^2. More exact evaluation of this factor can only be made by further experiment; it is recommended that every opportunity be utilized for obtaining additional data on natural frequency of actual ships. In the meantime, the matter will be studied by means of models.

W.P. Roop

TABLE II.

OBSERVED NATURAL FREQUENCY OKLAHOMA

Trial Instrument Position	1 Sperry For'd	2 Sperry For'd	2 Sperry Aft	2 Geiger For'd	3 Sperry For'd	3 Sperry Aft	3 Geiger For'd
	75.2	76.6	77.1	86.8	82.6		84.2
	74.4	79.0	80.2	85.4	78.8	74.4	83.9
	72.4	81.8	75.0	81.3	78.0	75.9	83.4
	71.2	82.4	77.3	83.1	78.2	74.9	83.6
	69.0	81.4		83.0	78.4	78.7	82.7
	70.4	84.2	Av.	83.3	79.4		84.1
	67.6	84.0	77.2	85.7	80.2	Av.	81.8
	70.2	81.4		87.6	79.6	76.0	86.3
	69.6	78.0		87.4	80.4		87.6
	66.4			90.1	79.4		85.8
	69.0	Av.		87.1	78		82.0
	68	80.1		86.9	78		84.9
	70			84.5	76		87.3
	75			84.4	74		82.9
	78			85.2	72		82.9
	87.4			84.4	72		85.3
	92			82.8	74		89.4
	86			86.6	72		86.2
	84			88.7	72		83.1
	78			86.8	76		86.4
				85.0	74		85.3
	Av.			86.3	80		84.5
	74.6			89.6	76		83.1
				85.5	76		81.6
				82.9	78		83.8
				82.1	76		83.5
				83.8	78		85.0
				84.4	76		
				84.4	78		Av.
				86.1	80		84.3
				85.5	78		
				89.4	78		
				Av.	Av.		
				85.2	77.5		

SUMMARY

Sperry:

1A	74.6
2F	80.1
2A	77.2
3F	77.5
3A	76.0
Av.	77.1
P.E.	0.8

Geiger:

2F	85.2
3F	84.3
Av.	84.75
P.E.	0.4

TABLE III.

SCHLICK'S CONSTANT

	CUYAMA		OKLAHOMA	
	Value	Ratio	Value	Ratio
Experimental	3.29	1.00	4.19	1.00
Calculated by Taylor's Method	4.47	1.36	4.40	1.05
Calculated by Lewis' Method	4.54	1.38	4.44	1.06
Calculated by Taylor's Method with Lewis' correction for virtual mass of water.	3.45	1.05	3.40	0.81

Note. - Assumed value of $E = 10,000 \frac{\text{tons}}{\text{in.}^2}$

APPENDIX.

DETAILED PROCEDURE FOR CALCULATING NATURAL FREQUENCY OF VERTICAL VIBRATION, FUNDAMENTAL MODE.

Column 1. Assumed Relative Amplitude. The values given are those of a "free-free" bar of uniform section.

Column 2. Weights, segregated by stations. The further course of the calculations follows the assumption that the weights in each station-length are uniformly distributed.

Column 3. Product of Columns 1 and 2. This is load in absolute force units, divided by φ, the acceleration per unit deflection. The net sum is the residual shear. If this differs from zero the amplitudes assumed in Col. 1 are not possible.

Column 4. To close the shear curve, adjust the amplitude curve by adding to each item in Col. 1 the proportional correction obtained by dividing the residual shear by the total weight, sum of column 2. Multiply this by items in Col. 2 to obtain corrections which, combined with Col. 3 give Col. 4.

Column 5. Summation of Col. 4. End value should now be zero. Net sum of this column gives residual Bending Moment.

Column 6. To close the Bending Moment curve the assumed amplitude must receive a second correction, zero amidships with linear increase on one side and decrease on the other. To find the amount of this correction assume first unit

correction varying from +1 at station 0 to -1 at station 20. In Col. 6 enter correcting loads obtained by multiplying the assumed correction to amplitude by Col. 2. The net sum gives residual shear, due to unit correction.

Column 7. Apply a correction similar to that in Col. 4 to obtain an adjusted unit correcting load giving zero residual shear.

Column 8. Summation of Col. 7, end value zero. The sum of Col. 8 gives Bending Moment due to unit correction assumed in Col. 6.

Column 9. Actual correction for residual Bending Moment is obtained by multiplying the unit corrections in Col. 8 by the ratio of actual residual Bending Moment from Col. 5 to the unit Bending Moment, sum of Col. 8. Applying this to Col. 5 gives shear fully corrected to give zero residuals in this and next column. Values are still in absolute units and still contain the factor $1/\varphi$.

Column 10. Summation of Co. 9. Corrected Bending Moment in tons x stations x $\frac{1}{\varphi}$. End value zero.

Column 11. Moment of Inertia in $\text{inch}^2\ \text{ft}^2$ of Transverse Section of Vessel at each Station, multiplied by Young's Modulus in tons/in^2 and by g., in units of 10^{10}.

Column 12. Col. 10 ÷ Col. 11. The total represents the difference of slope at the two ends of the deflection curve.

Column 13. Summation of Col. 12, starting with constant of integration equal to half the total of Col. 12. This will give a slope curve running from a negative value forward

through zero amidships to an equal positive value aft.

Column 14. Summation of Col. 13 giving numbers proportional to deflections. The end value is approximately zero.

Column 15. Add a linear correction to close Col. 14.

Column 16. For convenience in comparison, transfer the amplitudes in Col. 1 to the ends instead of the nodes as reference points.

Column 17. Col. 16 ÷ Col. 15. If the values are uniform, the amplitudes assumed represent a vibration consistent with the actual distribution of inertia and stiffness. If not, the nature of the departures from uniformity will suggest the changes necessary, which will consist in a flattening or accentuation of the peak or lateral shift of the nodes and not, of course, simply adjustments made in process of closure of shear and bending moment curves. In the example calculated below, with data from the CUYAMA, it was found necessary to make a lateral shift. After two trials the figures given in Col. 16a were chosen. Repeating the above process, somewhat abbreviated, gives the results in Col. 17a, which are acceptably uniform.

An acceptable mean value from Col. 17 having been found, it must be divided by the cube of the number of feet per station to convert length units in the three integrations from shear to deflection. The result is φ , acceleration per unit deflection, in reciprocal seconds squared. Desired frequency is $\frac{\varphi}{2\pi}$.

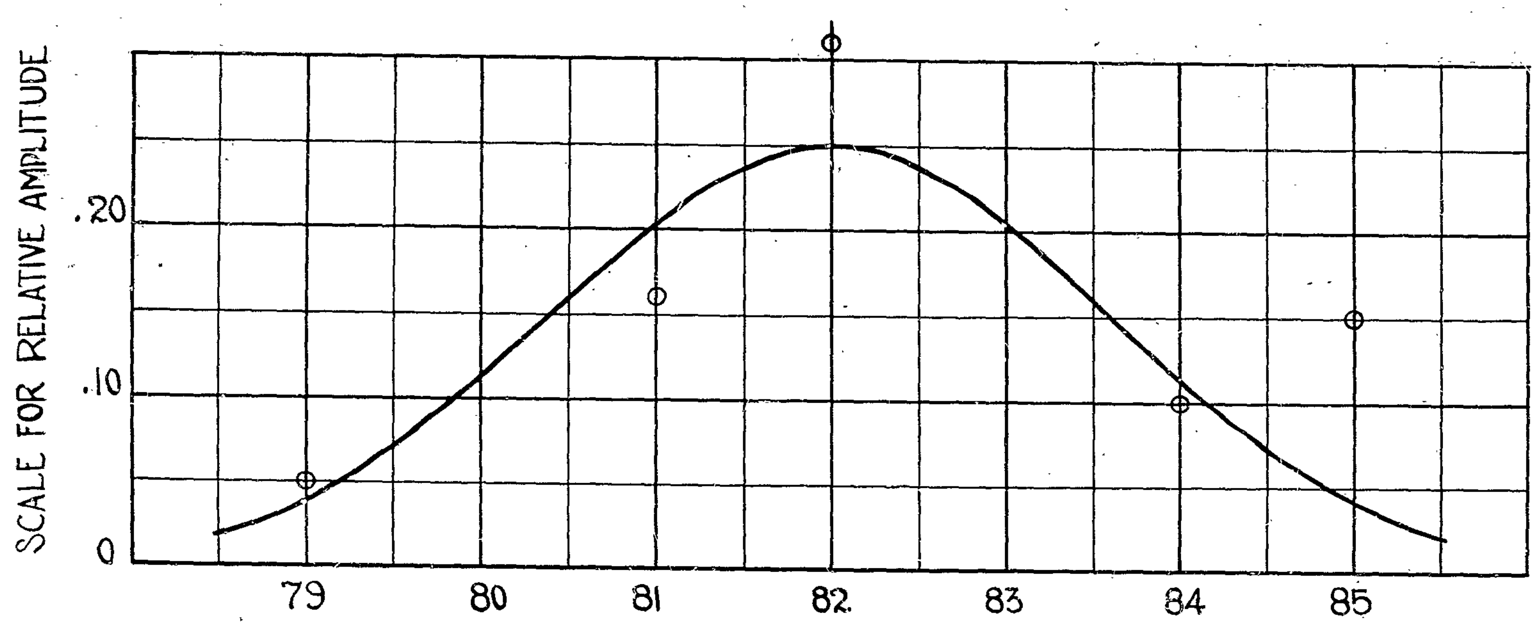
1	2	3	4	5	6	7	8	9	10	11	12	13
1.000	35	35	44	44	35	35	35	50	50	2.46	20.3	+2865
.768	200	152	204	248	180	181	216	284	334	4.93	67.8	+2845
.537	238	127	188	436	190	191	407	504	838	7.39	113.5	+2777
.313	321	100	183	619	225	227	634	724	1562	9.86	158.4	+2663
+.097	408	+ 39	144	763	245	248	882	909	2471	12.33	200.6	+2505
-.099	863	- 85	+138	901	431	436	1318	1119	3590	12.33	291.1	+2304
-.272	1219	-332	- 18	883	488	496	1814	1183	4773	12.33	387.2	+2013
-.414	1218	-504	-190	693	365	372	2186	1055	5828	12.33	472.8	+1626
-.521	1270	-662	-335	358	254	262	2448	763	6591	12.33	534.5	+1153
-.586	1225	-718	-402	- 44	122	130	2578	383	6974	12.33	565.5	+ 618
-.608	1221	-742	-427	-471	0	7	2585	- 43	6931	12.33	562.0	+ 52
-.586	1233	-722	-404	-875	-123	-115	2470	- 466	6465	12.33	524.5	- 510
-.521	1243	-648	-328	-1203	-249	-241	2229	- 834	5631	12.33	457.0	-1035
-.414	1095	-453	-171	-1374	-328	-321	1908	-1058	4573	12.33	371.0	-1492
-.272	593	-161	- 8	-1382	-2[illegible]	-233	1675	-1105	3468	12.33	281.2	-1863
-.099	595	- 59	+ 94	-1288	-298	-294	1381	-1059	2409	9.86	248.9	-2144
+.097	588	+ 57	209	-1079	-353	-350	1031	- 908	1501	7.39	203.0	-2393
.313	467	146	266	- 813	-327	-324	707	- 696	805	4.93	163.2	-2596
.537	233	125	185	- 628	-186	-185	522	- 542	263	2.46	107.0	-2759
.768	318	244	326	- 302	-286	-284	238	- 263	0	0		-2866
1.000	240	240	302	0	-240	-238	0	0				
	14823	-5086		-9459	-2627		27264					
		+1265		+4945	+2535							
		-3821		-4514	- 92							

$$\frac{3821}{14823} = 0.2578$$

14	15	16	17	16a	17a
2865	2677	232	86.7	216	80.5
5710	5334	463	86.8	435	81.4
8487	7923	687	86.7	651	82.0
11150	10397	903	86.9	862	82.7
13655	12714	1099	86.4	1059	83.0
15959	14830	1272	85.8	1234	82.9
17972	16655	1414	84.8	1383	82.6
19598	18093	1521	84.1	1499	82.3
20751	19057	1586	83.2	1575	82.0
21369	19487	1608	82.5	1608	81.8
21421	19351	1586	82.0	1598	81.8
20911	18652	1521	81.5	1545	81.9
19876	17430	1414	81.1	1445	81.9
18384	15750	1272	80.8	1311	82.2
16521	13699	1099	80.2	1140	82.1
14377	11367	903	79.4	944	81.9
11984	8784	687	78.2	723	81.0
9388	6000	463	77.2	491	80.5
6629	3054	232	76.0	248	79.9
3763	0				
			av. 82.65		av. 81.8
			n = 1.332		n = 1.325

FIGURE I

U.S.S. OKLAHOMA. RESONANCE CURVE



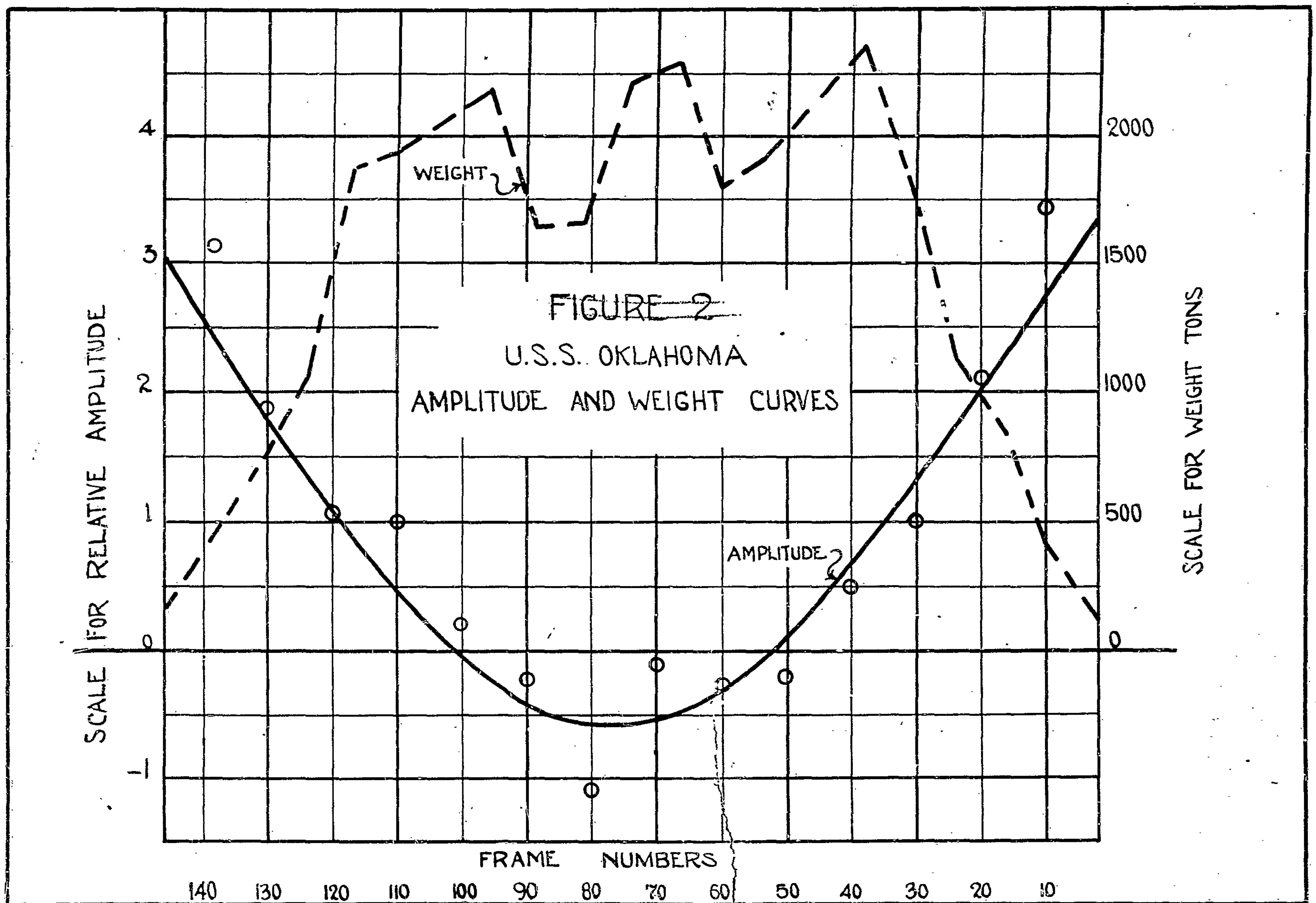


FIGURE 2
U.S.S. OKLAHOMA
AMPLITUDE AND WEIGHT CURVES
WEIGHT
AMPLITUDE
SCALE FOR RELATIVE AMPLITUDE
4
3
2
1
0
-1
SCALE FOR WEIGHT TONS
2000
1500
1000
500
0
FRAME NUMBERS
140
130
120
110
100
90
80
70
60
50
40
30
20
10